

How J M Keynes's Logical Theory of Probability Totally Refutes All Attacks on the Concept of Probability

Michael Emmett Brady

California State University, Dominguez Hills, College of Business Administration and Public Policy, Department of Operations Management, 1000 East Victoria St, Carson, California, 90747, USA

Abstract: J Derbyshire has recently repeated G L S Shackle's original attack on the concept of probability in a number of recent articles. He adds nothing new to Shackle's arguments of forty to eighty years ago that were completely refuted in 1959 by R. Weckstein, who based his refutation of Shackle on a relatively limited understanding of chapters 1-3 and 6 of Keynes's *A Treatise on Probability* (1921). Shackle never replied to Weckstein's repeated demonstrations in 1959 that Keynes's approach refuted his arguments, which held only against the limiting frequency, relative frequency, propensity and subjective interpretations of probability. Shackle's arguments failed completely when confronted with Keynes' s theory of logical probability.

J M Keynes's theory of probability is a logical, objective, epistemological approach that is based on partial ordering and not complete orderings. This results in non-additive, nonlinear interval valued probabilities, specified and operationalized in terms of logical propositions about any kind of event. It can easily deal with unique events, single events, crucial events, infrequent events, frequent events, non repeatable events, irreversible events, path dependence, sensitivity to initial conditions, emergence, complex causation, attractor states, partial uncertainty, complete and total uncertainty, fundamental uncertainty, irreducible uncertainty, by means of Keynes's weight of the evidence analysis, which, when combined with his interval valued probability, can demonstrate that Shackle's theory is a very special theory that applies only to situations of complete and total uncertainty, which Keynes categorized as ignorance. There is not a single one of Derbyshire's objections to the concept of probability left standing if Keynes's logical approach to probability is used.

Keywords: Keynes, J. M., Logical theory of probability, Interval valued probability, indeterminate-imprecise probability, weight of the evidence, partial order.

1. INTRODUCTION

J. Derbyshire has recently published four articles (see the references) reasserting the original claims made by G L S Shackle some forty to eighty years ago. In none of these articles does Derbyshire deal with R. Weckstein's original 1959 demonstration that, while Shackle's arguments held against other theories of probability, none of Shackle's arguments held against Keynes's Logical Theory of probability. Further, Derbyshire gives the very misleading impressions that, one, he is basing some of his articles on Keynes's work in the *A Treatise on Probability*(1921;TP) and the *General Theory*(1936;GT), and two, that Keynes's views on decision making under uncertainty and G L S Shackle's views are similar. In fact, Keynes completely rejected entirely G L S Shackle's completely subjective theory of probability and none of Derbyshire's results are based on the *A Treatise on Probability* (1921) and the *General Theory* (1936). In fact, Derbyshire's incorporation of Keynes in some his papers relies on one paragraph completely taken out of context from Keynes's February, 1937 reply to his critics in the *Quarterly Journal of Economics*. This is exactly what Shackle also did.

The paper will be organized in the following fashion. Section Two will present a summary of Keynes's logical theory of probability. Section three will present Derbyshire's list of what is claims is wrong with probability analysis. It is very similar to Shackle's earlier critique. Section Four will show why none of Derbyshire's criticisms apply to Keynes's theory. Shackle knew this back in 1938. Section Five will cover Derbyshire's failure to cover Keynes's GT and TP analysis of uncertainty and his strange reliance on one small paragraph from Keynes's 1937 *Quarterly Journal of Economics* article that was used by Shackle to avoid having to deal with the GT and TP. Section six will conclude.

2. THE OPERATIONAL CAPABILITY OF KEYNES'S LOGICAL THEORY OF PROBABILITY

The best summary of Keynes's logical, objective, epistemological approach to probability based on the work of George Boole was provided by F. Y. Edgeworth in 1922. We will use his statement as the focal point and expand on it as needed. Edgeworth's brief and concise statement below is the most powerful short summary of the TP ever written:

"Suffice it that many probabilities which are incapable of numerical measurement can be placed" between two numerical measures "(p. 32). We thus

*Address correspondence to this author at the California State University, Dominguez Hills, College of Business Administration and Public Policy, Department of Operations Management, 1000 East Victoria St, Carson, California, 90747, USA; E-mail: mandmbrady@juno.com

obtain the idea of a "finite probability "as one which exceeds some numerical probability (p. 257). Such measurements play a leading part in induction. To establish a generalization it is necessary that "with the experience we have" actually had there are finite probabilities, however small, derived "from some other source, in favour of the generalization" (p. 238). Round this nucleus of finite probability, through the operation of repetition and likeness, science grows. " An argument from "induction must always involve some element of analogy, and, 'on the other hand, few arguments from analogy can afford to "ignore altogether the strengthening influence of pure induction "[repetition]" (p. 255)." (Edgeworth, JRS, 1922, pp.107-108).

Edgeworth was able to figure out that Keynesian probabilities were mainly interval valued (indeterminate and imprecise) from a very careful and judicious reading of chapter III of the TP and his understanding of Keynes's use of Chebyshev's Inequality as a lower bound in Part V of the TP to model imprecision and the first "Safety First "approach to decision making in history.

The most important part of the TP is Part II. Great mathematicians, such as Edwin B Wilson, F Y Edgeworth, Emile Borel, Frank Ramsey, Ronald Fisher, and Bruno de Finetti, were unable to follow Keynes's mathematical, symbolic logic exposition in chapters 10-14 of the TP and Keynes's Boolean logic and algebra in chapters 15-17 of the TP. It was in these chapters that Keynes made profound breakthroughs that were only recognized by the author and the mathematician Theodore Hailperin in the 20th century. Chapters 10-14 of the TP clearly introduced the concept of non-additivity needed to explicitly support G. Boole's original, mathematical logic derivation of upper and lower probabilities on pp.265-268 of *The Laws of Thought* (LT; 1854), while at the same time providing the first axiomatic development of the laws of the probability calculus in history.

When it came to applying his approach, Keynes used his own modified, conditional probability approach to solving the interval valued probabilities of Boole from LT in chapters 15-17 of the TP under the name "approximation". Unfortunately, he used the name, "non-numerical probabilities", to describe the interval valued probabilities derived from the approximation analysis. This has led economists and philosophers into a quagmire where it is assumed that Keynes's operational approach is based on ordinal probability, not interval valued probability.

The following citations from chapter 15 of the TP were first referenced and analyzed by the author in his 1983 dissertation:

"The sphere of inexact numerical comparison is not, however, quite so limited. Many probabilities, which are incapable of numerical measurement, can be placed nevertheless *between* numerical limits. And by taking particular non-numerical probabilities as standards a great number of comparisons or approximate measurements become possible. If we can place a probability in an order of magnitude with some standard probability, we can obtain its approximate measure by comparison." (Keynes 1921, p.160; Keynes's emphasis).

This is identical to Keynes's summary of the issue of intervals in chapter III of the TP:

"A relation of probability does not yield us, as a rule, information of much value, unless it invests the conclusion with a probability which lies between narrow numerical limits." (Keynes, 1921, p.30) and,

"We frame two ideal arguments, that is to say, in which the general character of the evidence largely resembles what is actually within our knowledge, but which is so constituted as to yield a numerical value, and we judge that the probability of the actual argument lies between these two. Since our standards, therefore, are referred to numerical measures in many cases where actual measurement is impossible, and since the probability lies *between* two numerical measures, we come to believe that it must also, if only we knew it, possess such a measure itself." (Keynes, 1921, pp.31-32; Keynes's emphasis).

We can summarize Keynes's breakthrough now. First, both limiting frequency, relative frequency, propensity, subjectivist, and classical probability theories are special cases of Keynes's general, logical approach to probability. They are sound and valid within the specific, limited areas that they are applicable to. Second, non-additivity, not additivity, is the general case unless Keynes's weight of the evidence index, w , defined on the unit interval $[0, 1]$, equals, approaches or approximates 1, so that a decision maker has a complete set of all possible outcomes before he makes a choice. Probability preferences can be non-linear or linear. Keynes's modeling of w , the weight of the evidence, in his conventional coefficient of weight and risk, c , analysis on p.315 of the TP, also introduces non-additivity and non-linearity. Only a partial order can be defined.

Keynes followed Boole in using logical propositions about events to apply probability statements to. Thus, it is the probability of a proposition of an event, and not the probability of the event itself, that logical probability theorists use. In this format, ordinal probability, unique events, single events, irreversible events, crucial events, technological innovation and advance, obsolescence, unforeseen events, etc., are all easily analyzed using Keynes's logical approach to probability in the TP.

3. THE CLAIMS OF J. DERBYSHIRE REGARDING THE CONCEPT OF PROBABILITY

Derbyshire basically repeats the claims made by G L S Shackle in the time period from 1938-1992. Each of the four articles referenced is based on the attacks on probability originally made by Shackle. The reader can find an extensive list of Shackle's articles in each of the four Derbyshire articles listed in my references. Nowhere does R. Weckstein's refutation of Shackle's arguments, which were attacks on the limiting frequency, relative frequency, propensity, subjectivist, and classical probability theories foundation resting on assumptions of additivity, linearity, repetition, repeatability, complete orders, complete sample spaces, etc., mentioned. Weckstein, while acknowledging Shackle's points against the limiting frequency, relative frequency, propensity, subjectivist, and classical probability theories, pointed out that these attacks do not apply to Keynes's logical theory.

Derbyshire's papers are basically a repeat of Shackle's attack on the limiting frequency theory of probability and subjective theory of Ramsey, di Finetti, Savage and Friedman. I will use 2016(a) to cite from and list the corresponding sections of his other papers where he repeats his claims below.

Section 3.2.1 of Derbyshire repeats Shackle's attack on the inability of the limiting (relative) frequency interpretation of probability to deal with single (crucial) events because "By clearly establishing the simple concept of a serial, divisible experiment leading to a frequency-ratio Shackle can more easily outline an accurate conceptualisation of the opposite: a non-serial, no divisible experiment, which is not amenable to probabilistic reasoning. This is a single act which cannot be 'broken down into a number of more elementary performances' (Shackle, 1955a, p.23), all of which resemble each other and which, when taken together, allow the observer to accrue useful

knowledge in the form of a frequency-ratio. This is the nature of what Shackle calls a 'crucial experiment/decision' (Shackle, 1955a, p.6). Crucial decisions are incapable of absorption into a reference class by any means, even by pooling their occurrence across the many different individuals or organisations to which they occur; and, for this reason, probabilistic reasoning is of no use for dealing with crucial decisions"(Derbyshire, 2016. In press, pp.4-5).

Needless to say, Keynes's logical probability approach, as conceded by Shackle in 1976(1976, p.328), can be used for dealing with crucial decisions.

Section 3.2.2 repeats Shackle's attack on additivity in both the limiting frequency and subjective theories of probability:

"Under the Kolmogorov axioms, if the event space cannot be fully and precisely defined in advance, then the probability of any one outcome, or even of any subset of outcomes, cannot be defined. For most - or even all - crucial aspects of the future, a full set of possible outcomes is not known, and can never be known, in advance. We cannot know the full set of possible outcomes so as to reduce the problem of the future to one of probability (Shackle, 1984, p.75). When closure cannot occur, additional possible outcomes must be added as we feel our way into an opaque future. We must add to currently considered possible outcomes new possible outcomes, but this cannot be done within a probabilistic approach without diluting the importance attributed to currently considered outcomes. This problem of 'additivity' applies whether probability is frequency-based, as in a serial, divisible experiment, or conceived of in subjective terms, as in subjective expected utility theory."(Derbyshire, 2016 In press, p.5)

Both Boole's and Keynes's logical approaches to probability are non additive and non linear in the probability preferences expressed by decision makers. In special cases, where $w=1$, linearity and additivity can take place only as a special case.

Certain errors creep into Derbyshire's article when he asserts that "It is clear, then, that Shackle was not the first economist of note to distinguish between decisions that are crucial, and therefore subject to fundamental uncertainty for which probability is no aid, and other types of decision that are more akin to risk, which are more amenable to probabilistic reasoning.

Indeed, Shackle would have been more aware of the provenance of his ideas on uncertainty than anyone else since his second major contribution to economics after PST was his many writings on Keynes (Earl and Littleboy, 2014); Shackle was one of the 20th century's most prominent experts on Keynes (Earl and Littleboy, 2014).”(Derbyshire, 2016. In press, p.6).

Derbyshire is completely unaware that the definitions used by Keynes and Shackle defining uncertainty are completely different and conflicting. The claim made by Derbyshire that “Shackle was one of the 20th century's most prominent experts on Keynes” (Derbyshire, 2016, In press, p.6) is not supported by any evidence. Shackle constantly sought to avoid any direct engagement with any other academic who brought up the issue of the relevance of Keynes's work to Shackle's work. Shackle's favorite tactic was to simply not respond.

Finally, Derbyshire's claim that “While Knight and Keynes made a similar distinction between risk and uncertainty, only Shackle set out a complete, non-probabilistic approach to dealing with uncertainty. If any existing theory, then, can provide a theoretical underpinning for scenario planning - which also emphasizes the fundamental distinctiveness of uncertainty and how to deal with it - it must therefore be PST.”(Derbyshire, 2016. In press, p.7). is very dubious. He apparently is totally unaware of the work of Boole, Keynes and Hailperin.

The Keynes -Boole -Hailperin, indeterminate, imprecise, non additive approach to probability is superior to Shackle's possibility theory. Keynes had, in the early 1930's, already written on how the stockpiling of buffer stocks of crucial resources, such as oil, which had already experienced shocks earlier in the 20th century, before the 1973-75 and 1979-82 oil shocks, solved the potential problem of uncertainty in crucial resource inputs. It was the private oil companies themselves, who used their political and economic influence and lobbying power, to prevent the stockpiling of such crucial oil reserves that were responsible for the damage of the oil shocks. There could have been no significant shock if the USA has stockpiled 1-1.25 billion barrels, Europe 1-1.25 billion barrels, and Japan 1-1.25 billion barrels of oil.

I believe that Derbyshire is correct in pointing out the N N Taleb analysis based on anti fragility is a promising approach. However, this is a probability

based system. It simply is not based on the standard finite mean-finite variance distribution approaches, but on “infinite variance” distributions where the nice properties of the econometricians' multivariate normal distributions and artificial fixes, such as ARCH, GARCH, FIGARCH, etc. fall apart. Keynes's dispute with econometricians, such as Jan Tinbergen in the late 1930's, was not an attack on the concept of probability. It was an attack on the misuse of the limiting frequency approach to probability based on points Keynes had already made in 1908 and 1921, in his Second Fellowship dissertation and TP, respectively.

None of Derbyshire's attacks on the concept of probability apply to Keynes's logical approach. **None** of the points made by Derbyshire on pp.44-45, of Derbyshire, (2017a), concerning determinism, non stationarity, openness, additivity, crucial decisions, accurate aggregation or innate subjectivity, apply to Keynes 's logical approach to probability. None of Derbyshire's similar points made on pp.2-3 of (2017b) hold against Keynes's theory. Finally, none of the similar points, as analyzed above with respect to 2016a, that occur on pp.46-48 of Derbyshire (2016b), apply to Keynes.

4. DERBYSHIRE'S FAILURE TO COVER AND APPLY KEYNES'S TP AND GT APPROACH

Derbyshire claims that he is using Keynes's TP and GT as the foundation for his analysis in some of his articles. However, there is nothing in any of the four articles listed in my references where Derbyshire uses or cites Keynes at all from the TP or GT. What Derbyshire cites, correctly in one instance and incorrectly in another, is one small paragraph constantly cited by Shackle to “prove” that one did not have to read the TP or GT to understand Keynes. All one had to do, according to Shackle, was to read this one small paragraph in order to understand Keynes's concept of uncertainty:

“Keynes, writing in the 1930s (Keynes, 1936, 1937), made a similar distinction, describing his view on uncertainty thus: “By ‘uncertain’ knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable...The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence...*About these matters there is no scientific basis*

on which to form any calculable probability whatever" [emphasis added], (Keynes, 1937, p.113-114)." (Derbyshire, 2016a, section 4.1, p.5)

In fact, Keynes's definition of uncertainty in the GT is that Uncertainty is an inverse function of his weight of the evidence from the TP. See p.148 in chapter 12 of the GT.

I have already dealt with this issue of ignoring the GT and TP in the four 2017 SSRN papers referenced in my bibliography. However, from the above quotation from Keynes's 1937 reply in the *Quarterly Journal of Economics (QJE)*, it would be quite impossible to derive any of the conclusions derived in section 2 above concerning Keynes's approach toward using interval valued probability, non additivity, crucial, single, or unique events, etc.

Nor does Derbyshire improve in his 2017a paper. Derbyshire claims that, "Derbyshire (2016) shows that there were three early-to-mid twentieth century economists that emphasized a crucial distinction between risk and uncertainty, and who showed that probability has severe limitations in relation to the latter. Knight (1921) is perhaps the most widely-known. However, Keynes (1921, 1936) also made an important distinction between risk and uncertainty, stating that: 'By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable...The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence...About these matters there is no scientific basis on which to form any calculable probability whatever'." (Derbyshire, 2017a, p.46).

This is identical to the previous quotation and citation made by Derbyshire from the 1937 QJE article by Keynes. There are no citations to any of the references on Keynes in his Bibliography. There is no support for the claim that Keynes "... showed that probability has severe limitations in relation to the latter(uncertainty)" (Derbyshire, 2017a, p.44). Keynes used his interval valued approach to probability to deal with uncertainty, as well as his conventional coefficient of weight and risk, c , analyzed on p.314 of the TP. This quote has nothing to do with either the TP or GT or Vol. 14 of the CWJMK as listed on p.54 of Derbyshire (2017a). Derbyshire's overlooking of the TP and GT is similar to Hicks's overlooking of Keynes's IS-LM model

on pages 298-299 of Section IV of the GT in chapter 21.

CONCLUSIONS

Derbyshire's understanding of Keynes's logical theory of probability and approach to uncertainty is deficient. He bases his assessment of Keynes's views on one small paragraph taken from one page of Keynes's QJE reply. Derbyshire's claims about Keynes believing that the concept of probability could not be applied to uncertainty is incorrect. Keynes used his interval valued approach to probability to deal with uncertainty, as well as his conventional coefficient of weight and risk, c , analyzed on p.314 of the TP. Uncertainty means that the probabilities are non additive. Non additivity is the foundation for all of Keynes's work in decision theory and probability. Derbyshire's belief, that Knight, Keynes, and Shackle have similar views on uncertainty, is incorrect. Shackle's approach to uncertainty is that of a pure subjectivist who rejects all theories of probability, including Keynes's. Keynes's view is based on his objective, epistemological theory of logical probability. Keynes deals with uncertainty through his weight of the evidence analysis in the TP.

Derbyshire has no understanding about Keynes's work in probability. He is thus led to a view of the concept of probability that is incorrect because he ignores Keynes's logical theory of probability.

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